# Fundamentals of Robotics
## Inverse Kinematics - Problems

**Sergi Valverde**
*u1068016@correu.udg.edu*

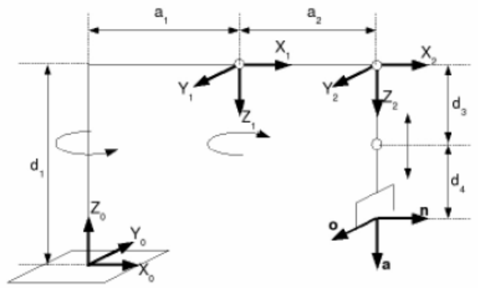
**Daniel Martínez**
*u1068321@correu.udg.edu*

May 8, 2011

## Problem 6

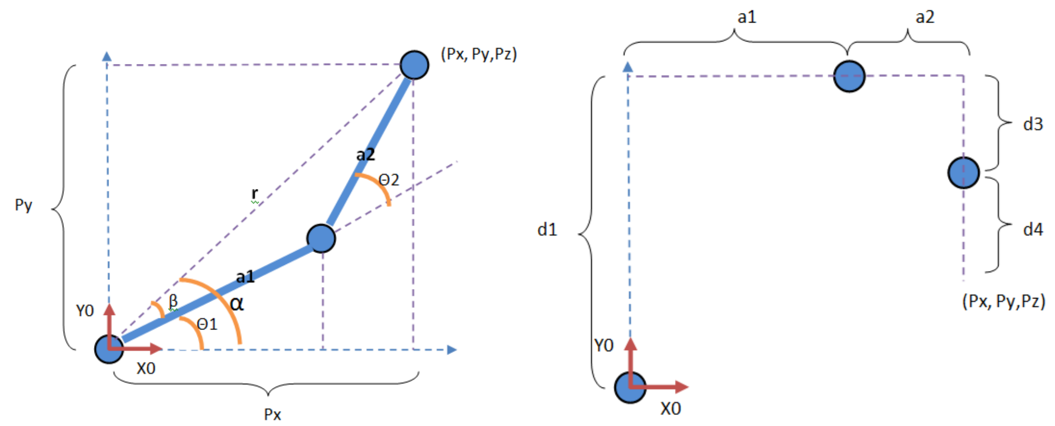
**Solve the IK problem for the following manipulator using the geometrical approach. The $^{R}T_{H}$ matrix is known. Note that you must clearly specify the different steps in order to obtain the results.**.

$$^{R}T_{H} = \begin{pmatrix} C_{1-2} & S_{1-2} & 0 & a_1 \cdot C_1 + a_2 \cdot C_{1-2} \\ S_{1-2} & -C_{1-2} & 0 & a_1 \cdot S_1 + a_2 \cdot S_{1-2} \\ 0 & 0 & -1 & d_1 - q_3 - d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$




## SOLUTION:

In order to solve the IK problem geometrical, we have represent the system as shown in Figure 1:



(a) Top view (b) Frontal view

Figure 1: Manipulator representation

The values that we want to find are the $\theta_1\theta_2$ and $q_3$. In this process, we use the cosine theorem (Figure 2 (b)) when it is necessary. In the first case, to arrange the $\theta_1$:

$$r = \sqrt{{P_x}^2 + {P_y}^2}$$

$$\alpha = atan2(P_y, P_x)$$

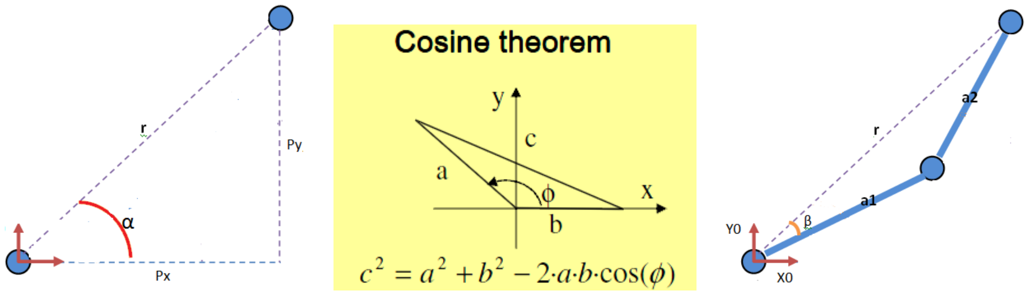
$$\theta_1 = \alpha - \beta$$

Using the cosine theorem from Figure 2 (b) over (c):

$${a_2}^2 = r^2 + {a_1}^2 - 2 \cdot r \cdot a_1 \cdot cos(\beta)$$

$$\beta = arcos(\frac{{a_2}^2 - r^2 - {a_1}^2}{-2 \cdot r \cdot a_1})$$

$$\boxed{\theta_1 = atan2(P_y, P_x) - arcos(\frac{{a_2}^2 - r^2 - {a_1}^2}{-2 \cdot r \cdot a_1}}$$




(a) (b) (c)

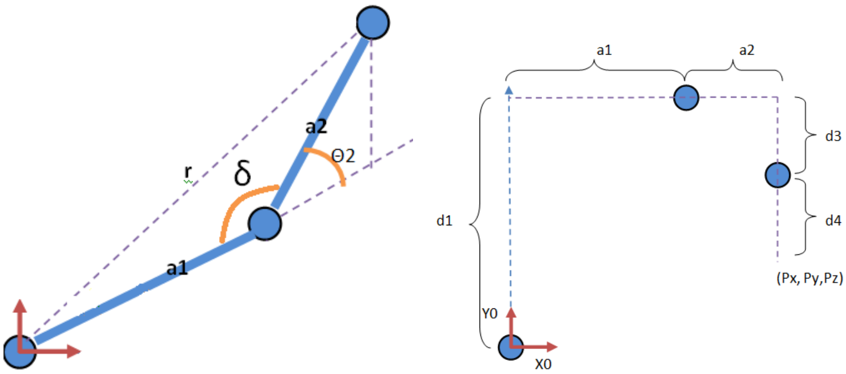
Figure 2

To arrange the value of $\theta_2$ we have to compute the $\delta$ angle using again the cosine theorem. Then, $\theta_2 = 180 - \delta$ (see Figure 3)

$$r^2 = {a_1}^2 + {a_2}^2 - 2 \cdot a_1 \cdot a_2 \cdot cos(\delta)$$

$$\delta = arcos(\frac{r^2 - {a_1}^2 - {a_2}^2}{-2 \cdot a_1 \cdot a_2})$$

$$\boxed{\theta_2 = 180 - arcos(\frac{r^2 - {a_1}^2 - {a_2}^2}{-2 \cdot a_1 \cdot a_2}}$$




(a) Find $\theta_2$ (b) Find $q_3$

Figure 3

In that case, to find $q_3$ is easier to use the symbolic solution. We can do it arranging the $q_3$ value in the $P_z$ value of the ${}^{R}T_{H}$ matrix. Also the geometric solution is shown in Figure 3 (b).

$$P_z = d_1 - q_3 - d_4$$

$$\boxed{q_3 = d_1 - P_z - d_4}$$

# Problem 8

**Given the development of the forward kinematics problem for the manipulator of the Figure** 4:

| Art. no. | θ | d | a | α | HOME |
|---|---|---|---|---|---|
| 1 | 0 | $q_1$ | $a_1$ | -90 | $d_1$ |
| 2 | $Q_2$ | 0 | $a_2$ | 90 | 0 |
| 3 | $Q_3$ | 0 | $a_3$ | 90 | 0 |

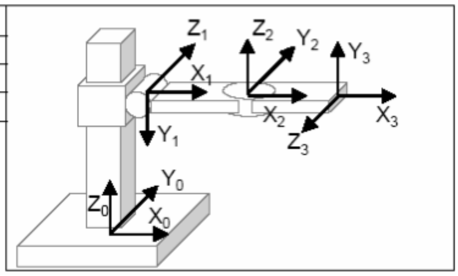



Figure 4: Denavit-Hartenberg

$$^0A_1 = \begin{pmatrix} 1 & 0 & 0 & a_1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad ^1A_2 = \begin{pmatrix} c_2 & 0 & s_2 & a_2 \cdot c_2 \\ s_2 & 0 & -c_2 & a_2 \cdot s_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad ^2A_3 = \begin{pmatrix} c_3 & 0 & s_3 & a_3 \cdot c_3 \\ s_3 & 0 & -c_3 & a_3 \cdot s_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^0A_3 = \begin{pmatrix} c_2 \cdot c_3 & s_2 & c_2 \cdot s_3 & a_3 \cdot c_3 \cdot c_2 + a_2 \cdot c_2 + a_1 \\ s_3 & 0 & -c_3 & a_3 \cdot s_3 \\ -s_2 \cdot c_3 & c_2 & -s_2 \cdot s_3 & q_1 - s_2 \cdot a_3 \cdot c_3 - a_2 \cdot s_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

**Solve symbolically the IK problem. What is the value of the configuration vector at the HOME position ?**

## SOLUTION:

Find $\theta_2$:

$$\boxed{q_2 = atan2(O_x, O_z)}$$

Find $\theta_3$:

$$\boxed{q_3 = atan2(N_y, -A_y)}$$

Find $\theta_1$: (Using $P_z$)

$$P_z = q_1 - s_2 \cdot a_3 \cdot c_3 - a_2 \cdot s_2$$

$$\boxed{q_1 = P_z + s_2 \cdot a_3 \cdot c_3 + a_2 \cdot s_2}$$

The configuration vector has the following form:

$$W = (P_x, P_y, P_z, e^{\frac{q_3}{\pi}} \cdot A_x, e^{\frac{q_3}{\pi}} \cdot A_y, e^{\frac{q_3}{\pi}} \cdot A_z)$$

In the home position $q_1 = d_1, q_2 = 0$ and $q_3 = 0$. So:

$$P_x = a_3 \cdot c_3 \cdot c_2 + a_2 \cdot c_2 + a_1 = a_3 \cdot cos(0) + a_2 \cdot cos(0) + a_1 = \mathbf{a_3 + a_2 + a_1}$$

$$P_y = a_3 \cdot s_3 = a_3 \cdot sin(0) = \mathbf{0}$$

$$P_z = q_1 - s_2 \cdot a_3 \cdot c_3 - a_2 \cdot s_2 = q_1 - sin(0) \cdot a_3 \cdot cos(0) - a_2 \cdot sin(0) = q_1 = \mathbf{d_1}$$

$$A_x = c_2 \cdot s_3 = cos(0) \cdot sin(0) = \mathbf{0}$$

$$A_y = -c_3 = -cos(0) = -\mathbf{1}$$

$$A_x = -s_2 \cdot s_3 = -sin(0) \cdot sin(0) = \mathbf{0}$$

Hence the $q_3 = 0$, the obtained configuration vector is:

$$W = (a_3 + a_2 + a_1, 0, d_1, e^{\frac{0}{\pi}} \cdot 0, e^{\frac{0}{\pi}} \cdot -1, e^{\frac{0}{\pi}} \cdot 0)$$

$$\boxed{W = (a_3 + a_2 + a_1, 0, d_1, 0, -1, 0)}$$